Angular Accel:

$$\theta = \left(\frac{S}{R}\right) \text{ rad}$$

$\theta_r \equiv \frac{S}{R}$ no units

$$\omega = \frac{\Delta\theta}{\Delta t} = \left(\frac{V}{R}\right) \text{ rad}$$

$\omega_r = \frac{V}{R}$ units $\frac{1}{S}$

spinning faster or slower

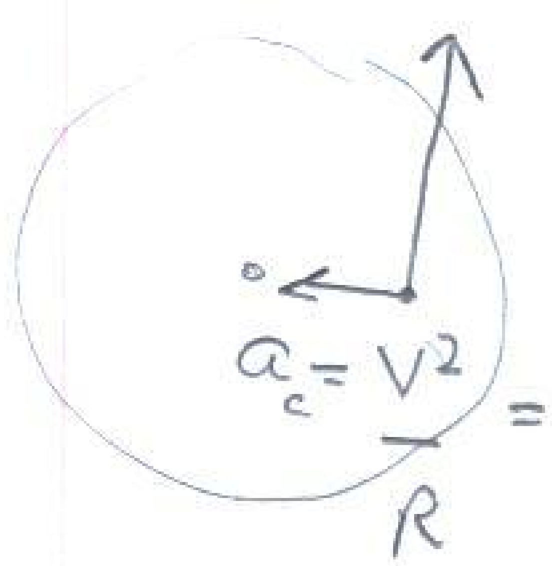
$$\alpha = \frac{\Delta\omega}{\Delta t} = \left(\frac{a_T}{R}\right) \text{ rad}$$

$\alpha_r = \frac{a_T}{R}$ units $\frac{1}{S^2}$

Constant Ang. Accel

$\omega = \omega_0 + \alpha t$	$V = V_0 + at$
$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x = x_0 + V_0 t + \frac{1}{2}at^2$
$\omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta$	$V^2 = V_0^2 + 2a\,\Delta x$

Picture of angular acceleration:



Problem a wheel starts from rest and accelerates to a speed of 12 rad/s in 3s.

a) find the angular Acceleration

b) For a bug 20 cm from the center find his position and acceleration at the end of 3s

Solution

a) $\alpha = \frac{\Delta \omega}{\Delta t} = \frac{12\,\text{rad/s}}{3\,s} = \frac{4\,\text{rad}}{s^2}$ $\qquad a_r = \frac{4}{s^2}$

b) $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

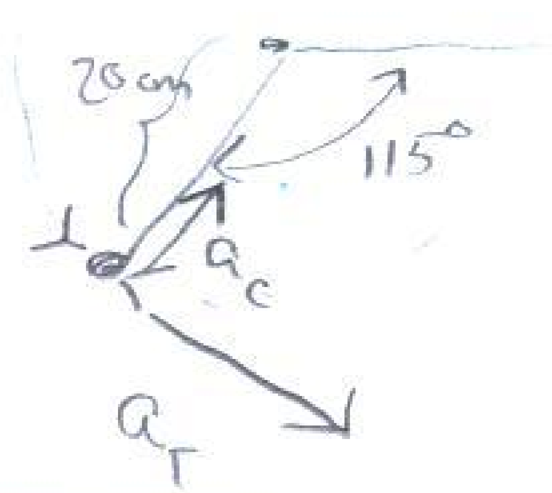
So $\theta(t=3s) = \cancel{\theta_0} + \cancel{\omega_0} t + \frac{1}{2}\left(4\,\frac{\text{rad}}{s^2}\right)(3s^2)$

$\theta(t=3s) = 18\,\text{rad} \implies \theta = 2.68\,\text{rev} = 2\,\text{rev} + 244.8^\circ$

c) $a_T = \alpha_r r = \left(4\,\frac{1}{s^2}\right) \cdot 0.2\,m = 0.8\,m/s^2$

$a_c = \frac{v^2}{R} = \omega_r^2 R = \left(12\,\frac{1}{s}\right)^2 \cdot (0.2\,m) = 28.8\,m/s^2$

Graph



$a = \sqrt{a_c^2 + a_T^2} = 28.81\,m/s^2$

Kinetic Energy

$$KE = \frac{1}{2} m v^2$$

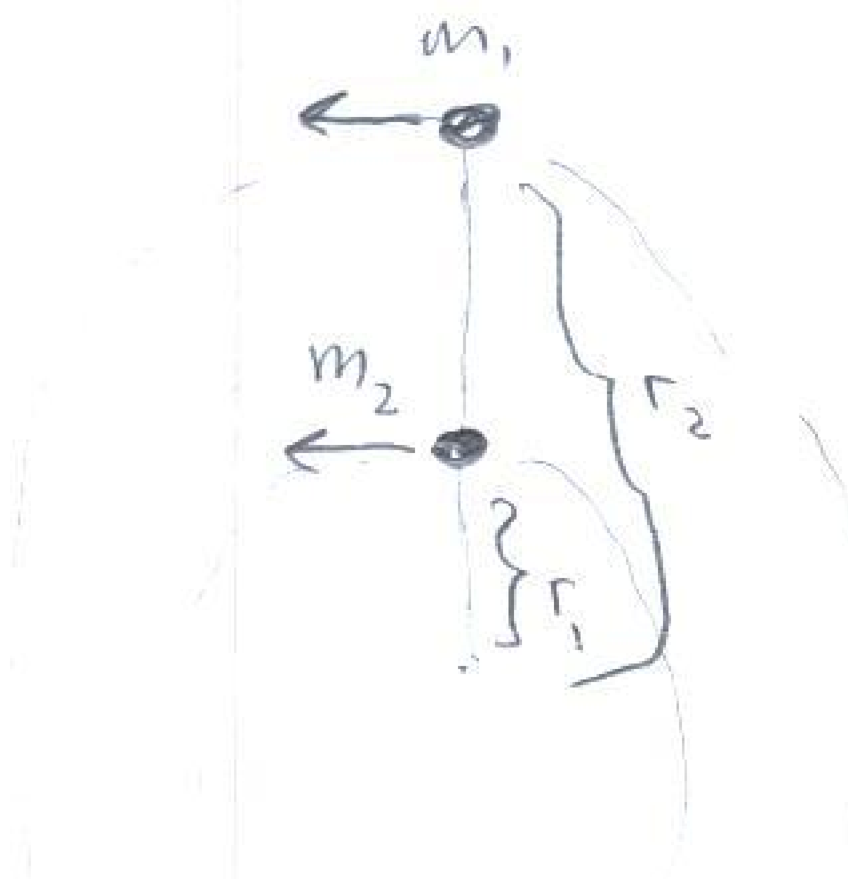
$$KE = \frac{1}{2} m (\omega_r R)^2$$

$$KE = \frac{1}{2} \underbrace{(m R^2)}_{\equiv\, I} \omega_r^2 = \frac{1}{2} I \omega_r^2$$

$I \equiv mR^2$ (one object)

moment of inertia

Two Objects



$$KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$KE = \frac{1}{2} m_1 (\omega_r r_1)^2 + \frac{1}{2} m_2 (\omega_r r_2)^2$$

$$KE = \frac{1}{2} m_1 r_1^2 \omega_r^2 + \frac{1}{2} m_2 r_2^2 \omega_r^2$$

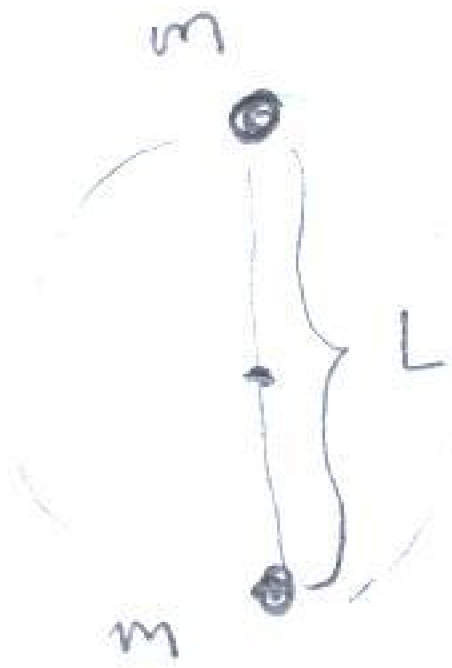
$$= \underbrace{\frac{1}{2}\left[m_1 r_1^2 + \frac{1}{2} m_2 r_2^2\right]}_{\equiv\, I} \omega_r^2$$

$I = \sum_i m_i r_i^2$ (many objects)

$$I = M \frac{\sum m_i r_i^2}{\sum m_i} = M \langle r^2 \rangle_M$$

Moment of Inertia depends on axis of rotation

Ex: Find the moment of inertia about the center



Solution:

$$\frac{I}{M_{TOT}} = \frac{\sum m_i r_i^2}{\sum m_i} = \frac{m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2}{m+m} = \frac{2\left(\frac{L}{2}\right)^2}{2}$$

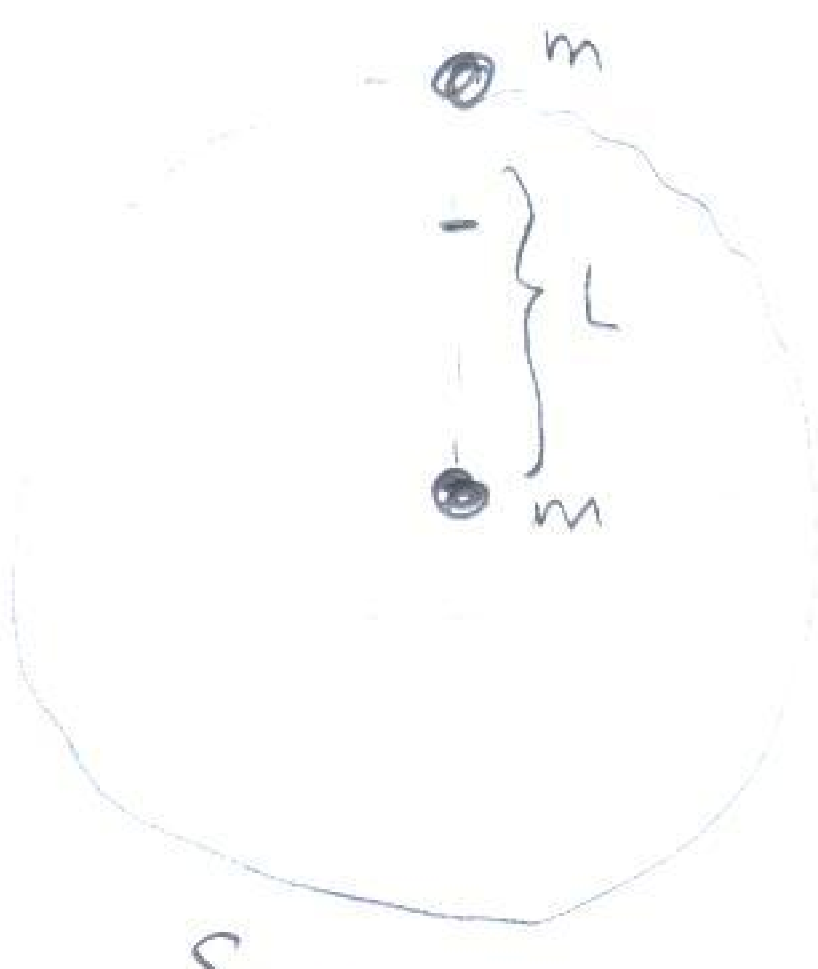
$M_{TOT} = 2m$

$$= \left(\frac{L}{2}\right)^2$$

This is "obvious" $\frac{I}{M_{TOT}} = \langle r^2 \rangle_M = \left(\frac{L}{2}\right)^2$

$$= \frac{L^2}{4}$$

Next Consider



$$\frac{I}{M_{TOT}} = \frac{\sum m_i r_i^2}{\sum m_i} = \frac{m L^2 + m \cdot 0}{m+m} = \frac{L^2}{2}$$

$$= \frac{L^2}{2}$$

Summary

$$\frac{I}{M_{TOT}} = \frac{\sum m_i r_i^2}{\sum m_i} = \frac{\int dm\, r^2}{\int dm}$$

$$\boxed{KE = \frac{1}{2} I \omega_r^2}$$

Next Consider the PE of continuous body

$$PE_g = \sum m_i g y_i = g \sum m_i y_i = Mg \frac{\sum m_i y_i}{\sum m_i}$$

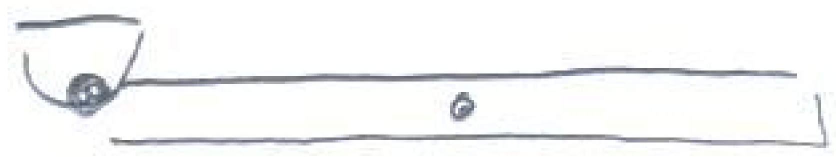
$$PE_g = Mg\, y_{cm}$$

The potential energy of a continuous body is simply

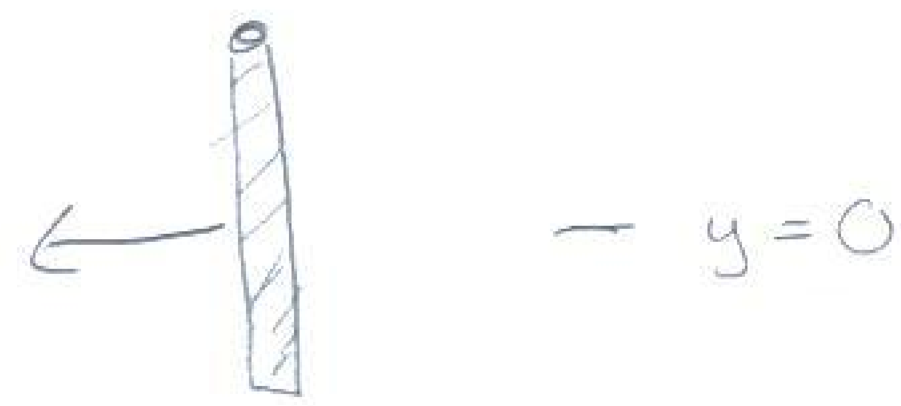
Mass $\times$ g $\times$ (height of center of mass)

Example

Before



After



A rod of Length L is released, find its angular speed at the bottom of the arc:

Suppose $L = 1m$

Solution

$$\cancel{W_{ext}}^{\,0} = \Delta KE + \Delta PE$$

$$\cancel{KE_i}^{\,0} + PE_i = KE_f + PE_f$$

$$Mg\,y_{cm} = \frac{1}{2} I \omega_r^2 + 0$$

$$\cancel{M} g \frac{L}{\cancel{2}} = \frac{1}{\cancel{2}}\left(\frac{1}{3}\cancel{M}L^2\right)\omega_r^2$$

$$\frac{3g}{L} = \omega_r^2$$

$$\sqrt{\frac{3g}{L}} = \omega_r$$

$$\sqrt{\frac{3 \times 9.8\,m/s^2}{1m}} = \omega_r$$

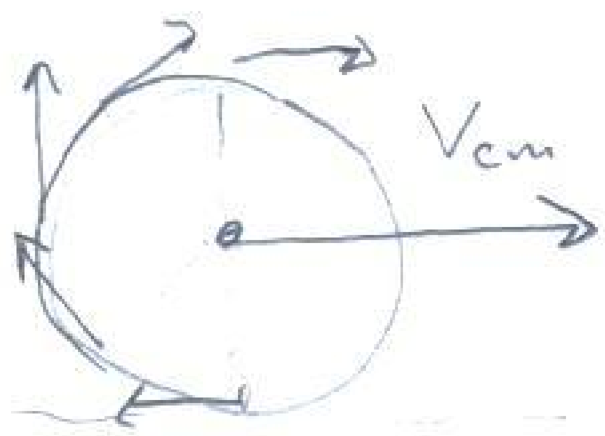
$$5.4\ 1/s = \omega_r$$

$$\omega = 5.4\ \frac{rad}{s}$$

$$\omega = \frac{5.4}{2\pi}\ \frac{rev}{s}$$

$$\omega = 0.86\ \frac{rev}{s}$$

Rolling without Friction



Minus because this arrow points in opposite direction

$$V_{cm} = \frac{\Delta x}{\Delta t}$$

① $V_{bottom} = V_{cm} - \underbrace{R\omega}_{V_{bottom}\ \text{rel. to cm}}$

$$= V_{cm} - R\frac{\Delta\theta}{\Delta t}$$

$$V_{bottom} = V_{cm} - \frac{\Delta x}{\Delta t} = V_{cm} - V_{cm} = 0$$

② $V_{top} = V_{cm} + V_{top\text{-}rel\text{-}cm}$

$$V_{top} = V_{cm} + R\omega$$

$$V_{top} = V_{cm} + R\frac{\Delta\theta}{\Delta t} = V_{cm} + V_{cm} = 2V_{cm}$$

The kenetc evergy of a rolling object ?

$$KE = \sum \frac{1}{2} m_i v^2$$

$$KE = \frac{1}{2} \sum m_i (v - v_{cm} + v_{cm})^2$$

$$KE = \frac{1}{2} \sum m_i (v - v_{cm})^2 + \frac{1}{2} \sum m_i 2v_{cm} (v - v_{cm}) + \frac{1}{2} \sum m$$

$$KE = \frac{1}{2} \sum m_i (v - v_{cm})^2 + (\sum m_i v - M v_{cm}) v_{cm} + \frac{1}{2} M$$

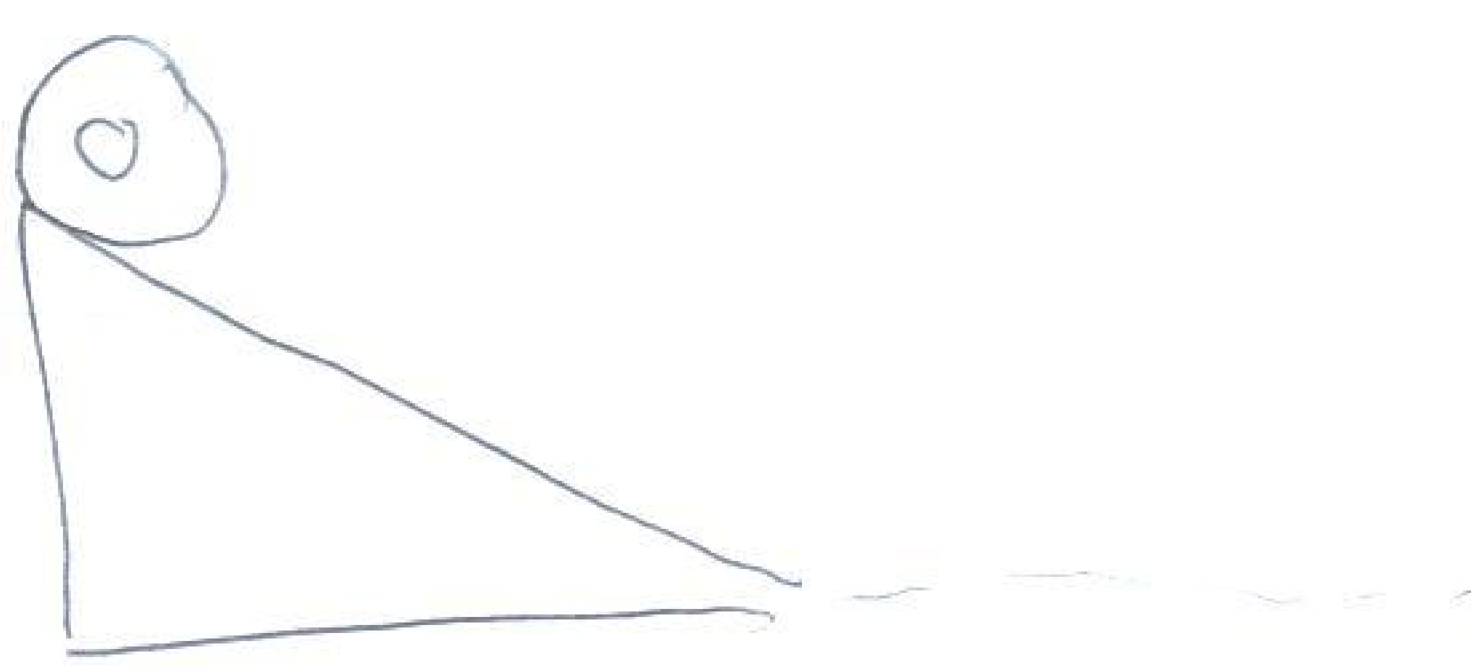
$$KE = KE_{rel\ cm} + \cancel{(M v_{cm} - M v_{cm}) v_{cm}} + \frac{1}{2} M$$

$$\boxed{\begin{aligned} KE &= \underbrace{KE_{rot}} + KE_{cm} \\ KE &= \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2 \end{aligned}}$$

Calchlate the speed of a hollow cylinder as it rolls down the hill of height 0.5m of radius 10 cm

$I = MR^2$ from Pg. 3

$0.5m = h$



$$\cancel{W_{ext}}^{0} = \Delta KE + \Delta PE$$

$$\cancel{KE_i}^{0} + PE_i = KE_f + \cancel{PE_f}^{0}$$

$$Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega_r^2 \qquad v_{cm} = \omega_r R$$

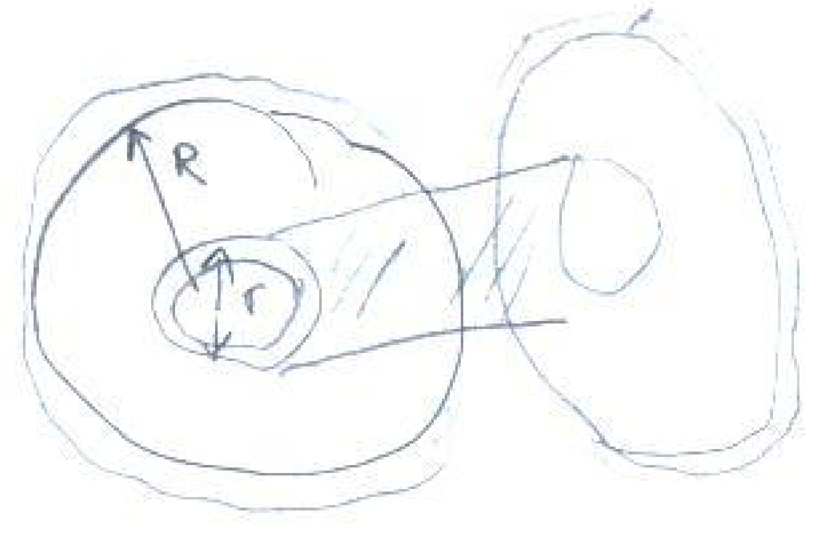
$$\cancel{M}gh = \frac{1}{2}\cancel{M}v_{cm}^2 + \frac{1}{2}(\cancel{M}R^2)\left(\frac{v_{cm}}{R}\right)^2$$

$$gh = \frac{1}{2}v_{cm}^2 + \frac{1}{2}v_{cm}^2$$

$\sqrt{gh} = v_{cm}$ ← This is less than the free fall speed because some of the PE is stored in rotation

Example using Tables of Momet of Inertia

- The moment of Inertia of a compound object is a sum of the moments of inertia



Reason: $I = \int dm\ R_{\perp}^2$

Integral is a glori[fied] sum.

<u>Problem</u>

A spool consists of two outer rims (radius ... of mass 20g and hollow inner cylinder of radius $r_0 = 2cm$ and mass 100g.
Find I

$$I_{tot} = 2I_{rim} + I_{cylnder}$$

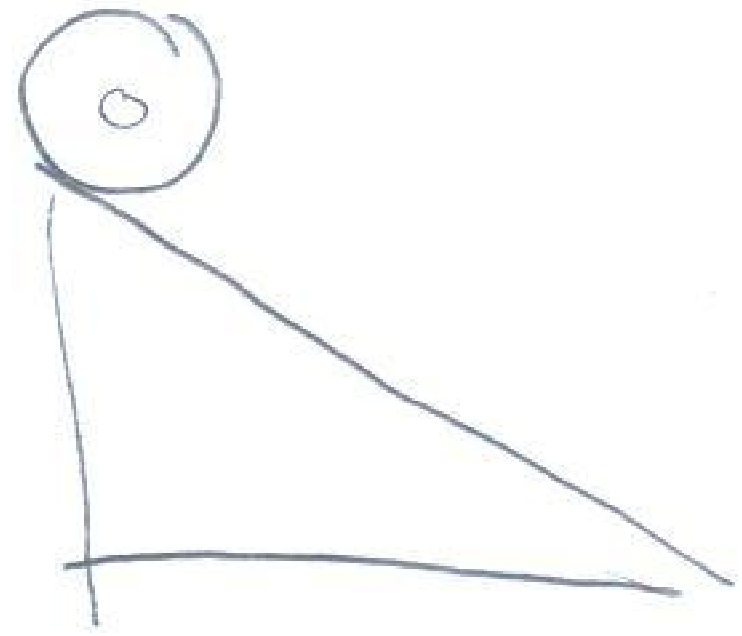
$$I_{tot} = 2\cdot\left(\frac{1}{2} m_{rim} R_{rim}^2\right) + M_{cyl} R_{cyl}^2$$

from pg. 304 (c) & (a) respect[ively]

$$I_{tot} = m_{rim} R_{rim}^2 + m_{cyl} R_{cyl}^2$$

$$= (0.02\,kg)(0.05\,m)^2 + (0.1\,kg)(0.02\,m)^2$$

$$I_{tot} = 9\times10^{-5}\ kg\,m^2$$



Find the speed of this object as it reaches the bottom

$$\cancel{KE_i}^{0} + PE_i = KE_f + \cancel{PE_f}$$

$$Mgh = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \omega^2$$

$$Mgh = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \frac{v_{cm}^2}{R^2}$$

$$2gh = v_{cm}^2 + \left(\frac{I}{MR^2}\right) v_{cm}^2$$

$$2gh = \left(1 + \frac{I}{MR^2}\right) v_{cm}^2$$

$$\sqrt{\frac{2gh}{1 + I/MR^2}} = v_{cm}$$

$$MR^2 = (0.02 + 0.10)\,kg \times (.05m)^2$$

$$MR^2 = 3 \times 10^{-4}\ kg\,m^2$$

So

$$\frac{I}{MR^2} = 0.333$$

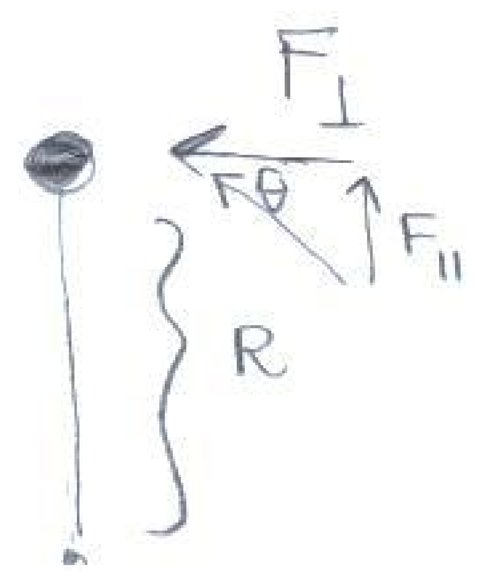
What makes things spin:

- Analaguous to what makes things move

Answer: Force perpendicular to moment arm:

$$\underbrace{\tau}_{\text{Torque}} = F_{\perp} R = F_{\phi} R \sin\theta$$

Proof:



$$\sum F_{\perp} = m a_{\perp}$$

$$F_{\perp} = m \alpha_r R$$

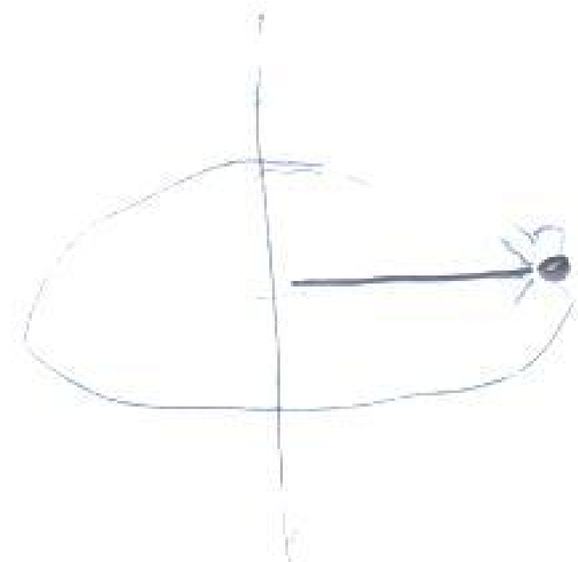
$$F_{\perp} R = m \alpha_r R^2$$

$$\underbrace{F_{\perp} R}_{\text{Torque}} = \underbrace{(mR^2)}_{\equiv I \text{ like rotational mass}} \underbrace{\alpha_r}_{\text{angular acceleration}}$$

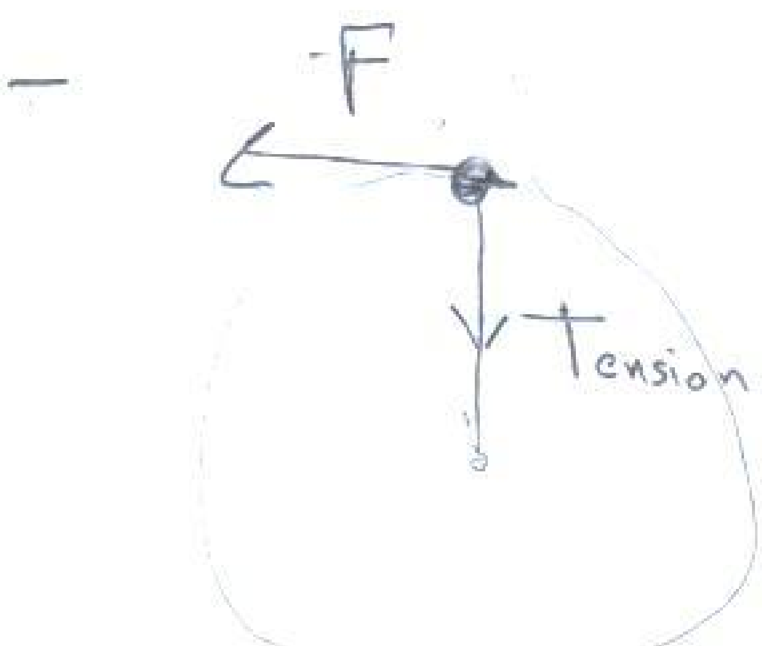
" F = m a "

$$\boxed{\tau = I\alpha}$$

Problem: A toy plane provides a net thrust of 0.800 N, The airplane weighs 0.2 kg and flies in a 0.8m horiztal circle attached by wire to a pole



Find how long it takes to make one completer revolution:



(1) Find α

$$\sum \tau = I\alpha_r$$

$$\tau_{tension} + \tau_F = I\alpha_r$$

See next page

$$FR = mR^2 \alpha_r$$

$$\frac{F}{mR} = \alpha_r$$

(2) Angle in radians as a function of t

$$\theta_r = \frac{1}{2}\alpha_r t^2$$

We want $\theta = 2\pi$ radia

or $\theta_r = 2\pi$

$$2\pi = \frac{1}{2}\frac{F}{mR} t_*^2$$

special t when θ_r

$$\sqrt{\frac{4\pi m R}{F}} = t_*$$

$t_* = 1.53$

Angular Accel:

$$\theta = \left(\frac{S}{R}\right) \text{ rad}$$

$$\theta_r \equiv \frac{S}{R} \qquad \text{no units}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \left(\frac{V}{R}\right) \text{ rad} \qquad \omega_r = \frac{V}{R} \qquad \text{units } \frac{1}{s}$$

spinning faster or slower

$$\alpha = \frac{\Delta\omega}{\Delta t} = \left(\frac{a_T}{R}\right) \text{ rad} \qquad \alpha_r = \frac{a_T}{R} \qquad \text{units } \frac{1}{s^2}$$

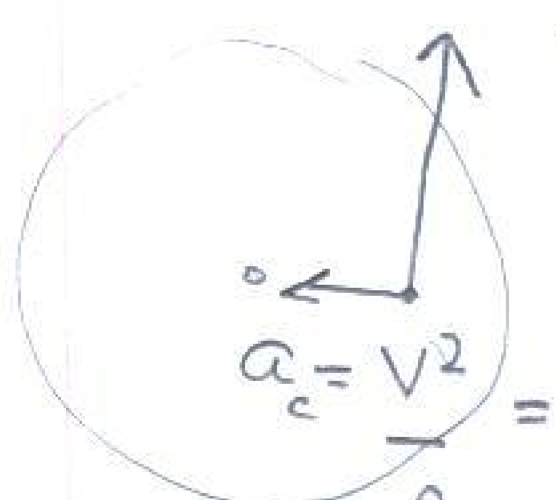
Constant Ang. Accel

$$\omega = \omega_0 + \alpha t \qquad V = V_0 + at$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \qquad x = x_0 + V_0 t + \frac{1}{2}at^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta \qquad V^2 = V_0^2 + 2a\,\Delta x$$

Picture of angular acceleration:



$$a_T = \alpha R$$

$$a_c = \frac{V^2}{R} = \omega^2 R$$